Blaschke's Curvature Energies and Minimal Tori in $\mathbb{S}^3$

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Abstract

We introduce a curvature energy functional acting on planar curves of $\mathbb{S}^3$ which extends a functional studied by Blaschke. Based on a technique involving Killing vector fields, we show the existence of a biparametric family of closed critical curves for this functional.

Next, using these closed critical curves as generators, we describe two constructions of surfaces in $\mathbb{S}^3$, Hopf tori and binormal evolution tori, which give rise to closed tori critical for a Blaschke's type variational problem over surfaces and minimal tori of $\mathbb{S}^3$, respectively.

Finally, some properties of the critical generating curves are used to obtain results about the tori.