Conformal Geometry and its Application to Neurogeometry

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Le seguenti lezioni (di carattere elementare), di circa 1h20min ciascuna, sono rivolte a dottorandi, assegnisti e a tutto il personale docente. Durante le lezioni saranno discusse anche possibili linee di ricerca.

Aula: 15S del Politecnico di Torino

Orario: 15.00–16.30

Lecture I –08/06/2015–
Presentation of the course. Conformal Geometry of the sphere.
Minkowski space and Lorentz group. Three orbits of the Lorentz group $SO_{1,3}$ in the Minkowski space and associated geometries. Conformal sphere as the projectivised light cone (celestial sphere) and as the flag manifold. Classification of 1-parameter subgroups of the Lorentz group and the homogeneous curves in the conformal sphere. Conformal circles and their description.

Lecture II –10/06/2015–
An introduction to conformal geometry of manifolds
Conformal structure on a manifold. The aim of the conformal geometry. Conformal geometry of curves. Conformally homogeneous manifolds. Prolongation of the linear conformal Lie algebra $\mathfrak{co}(V)$ and the prolongation of the conformal structure. Cartan connection of a conformal structure as the bundle of second order frames. Conformal circles. Relation between conformal circles and Riemannian geodesics. Weyl curvature tensor and conformally flat manifolds. Application to classical mechanics.

Lecture III –15/06/2015–
Architecture of visual system
Basic principles of the construction of visual system. Eye as an optical device and the central projection of a surface to retina. Eye as a rotating rigid body. Neurons as functionals. Processing of the information in retina and LGN. Problem of the conformal invariancy (stability problem).Architecture of the primary visual cortex VI and pinwhee l field.

Lecture IV –17/06/2015–
Models of visual systems in early vision
Visual cortex VI as a fibre bundle (Hubel-Wiesel). Problem of internal parameters. Petitot model of VI cortex as a Lie contact bundle. Petitot-Citti-Sarti model of VI cortex as a symplectic bundle. Hypercolumns. Bressloff-Cowan model of hypercolumns as a sphere. Unification of Petitot-Citti-Sarti and Bressloff-Cowean models : VI cortex as the bundle of second order conformal frames (Cartan connection) of the conformal sphere. Application to solutions of stability problems.