POLITECNICO DI TORINO

DISMA Dipartimento di Scienze Matematiche G. L. Lagrange

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Online seminar

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Hosted on: Zoom

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The Kirchhoff-Plateau problem and its generalizations

Prof. Lussardi introduces the seminar.

Abstract

The Kirchhoff-Plateau problem concerns the equilibrium shapes of a system in which a flexible filament in the form of a closed loop is spanned by a liquid film, with the filament being modeled as a Kirchhoff rod and the action of the spanning surface being solely due to surface tension. Giusteri, Lussardi and Fried in [6] established the existence of an equilibrium shape that minimizes the total energy of the system under the physical constraint of non-interpenetration of matter, but allowing for points on the surface of the bounding loop to come into contact. In [2, 3], we use this result to generalize the situation studying a system composed by several rods linked in an arbitrary way and tied by a soap film and we perform some experiments to validate our result.

We also study the Plateau problem, i.e. the boundary is an elastic curve. In [4], we obtain the minimal energy solution of the Plateau problem with elastic boundary as a variational limit of the minima of the Kirchhoff-Plateau problems with a rod boundary when the cross-section of the rod vanishes. The limit boundary is a framed curve that can sustain bending and twisting. Finally, since computing the minimum of the energy functional is quite easy, in [5] and [1], we study the critical points of the Plateau problem to characterize its entire mechanical structure and to compute all the equilibrium configurations. First, we propose a slight variation of the Lagrange multiplier theorem in infinite dimension, obtaining a non-homogeneous first order differential system of equations [5]. Then, in [1], using an induction argument, we construct infinitely many critical points for a functional depending only on the curvature and both on the curvature and the torsion.

[1] G. Bevilacqua, A. De Rosa, L. Lussardi, Critical points of the Plateau problem, in preparation.
[2] G. Bevilacqua, L. Lussardi, A. Marzocchi, Soap film spanning electrically repulsive elastic protein links, Proceedings of School & Research Workshop Mathematical Modeling of Self-Organizations in Medicine, Biology and Ecology: from micro to macro, Atti Accad. Peloritana Pericolanti Cl. Sci. Fis.Mat. Natur. 96 (2018), suppl. 3, A1, 13pp.
[3] G. Bevilacqua, L. Lussardi, A. Marzocchi, Soap film spanning an elastic link, Quart. Appl. Math. 77 (3) (2019), 507 - 523.
[4] G. Bevilacqua, L. Lussardi, A. Marzocchi, Dimensional reduction of the Kirchhoff-Plateau problem, to appear on J. of Elasticity.
[5] G. Bevilacqua, L. Lussardi, A. Marzocchi, Variational analysis of inextensible elastic curves, in preparation.
[6] G.G. Giusteri, L. Lussardi, E. Fried, Solution of the Kirchhoff-Plateau problem, J. Nonlinear Sci. 27 (2017), 1043 - 1063.

Biography

Giulia Bevilacqua received her B.Sc. and M.Sc. degree in Mathematics from Universitá Cattolica del Sacro Cuore, Brescia, in 2015 and 2017, respectively. In 2017, she joined Politecnico di Milano as a Ph.D. student.

Her research interests concern, on one hand, mathematical and physical modeling of soft biological tissues, in particular in shape instabilities and mechanical phenomena in the embryo, and the Plateau problem (i.e. existence of a surface with minimal area that spans a given boundary) on the other.

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