NON-PARAMETRIC INFORMATION GEOMETRY WITH DERIVATIVES

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Non-parametric Information Geometry according to a series of papers starting with [6] consists of a manifold on the set of positive densities of a measure space. The manifold is modeled on the Banach space of exponentially integrable random variables. In a more recent presentation [4] the relevant structure is described a Banach bundle of couples (p, u) where p is a positive density and u is a random variable such that $E_p(u) = 0$. Each connected component of the base manifold, consisting of densities which are connected by an open exponential family, is fully described in [7]. Other methods for dealing with the infinite-dimensional geometry of probabilities are available, in particular [1]. The main limitation of this approach is the inability to deal with properties of the statistical models depending on the structure of the sample space where the densities are defines e.g., the smoothness. In the framework of Gaussian spaces [2] it is actually possible to study such properties while retaining the same bundle structure. Preliminary results have been published in [3, 5] and further research is in progress. An example of application is the study of Hyvärinen divergence [2].

References

1. Nihat Ay, Jürgen Jost, Hông Vân Lê, and Lorenz Schwachhöfer, *Information geometry*, Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], vol. 64, Springer, Cham, 2017. MR 3701408
2. Aapo Hyvärinen, *Estimation of non-normalized statistical models by score matching*, J. Mach. Learn. Res. **6** (2005), 695–709. MR 2249836
3. Betrand Lods and Giovanni Pistone, *Information geometry formalism for the spatially homogeneous Boltzmann equation*, Entropy **17** (2015), no. 6, 4323–4363.
4. Giovanni Pistone, *Nonparametric information geometry*, Geometric science of information (Frank Nielsen and Frédéric Barbaresco, eds.), Lecture Notes in Comput. Sci., vol. 8085, Springer, Heidelberg, 2013, First International Conference, GSI 2013 Paris, France, August 28-30, 2013 Proceedings, pp. 5–36. MR 3126029
5. ———, *Information geometry of the Gaussian space*, arXiv:1803.08135, 2018.
6. Giovanni Pistone and Carlo Sempi, *An infinite-dimensional geometric structure on the space of all the probability measures equivalent to a given one*, Ann. Statist. **23** (1995), no. 5, 1543–1561. MR 97j:62006
7. Marina Santacroce, Paola Siri, and Barbara Trivellato, *New results on mixture and exponential models by Orlicz spaces*, Bernoulli **22** (2016), no. 3, 1431–1447. MR 3474821

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